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Weakly Generalized β- Continuous Mapping in Neutrosophic Bitopological Spaces

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Abstract: This paper explores the notion of β-continuity in neutrosophic bitopological spaces, a specialized area of mathematics that extends classical topological concepts to handle indeterminate or uncertain information. The study begins with the introduction of $\tau_1\tau_2$ semi-closed sets and $\tau_1\tau_2$-weakly continuous functions, which are fundamental building blocks. Key results include Proposition 2.1.3, which characterizes $\tau_1\tau_2$-weakly β-continuous mappings in terms of pre-images and β-interiors of open sets in the codomain space. Propositions 2.1.4 and 2.1.5 establish equivalent conditions for $\tau_1\tau_2$-weakly β-continuous functions involving pre-images, closures, and regular closed sets. Propositions 2.1.6 and 2.1.7 provide alternative characterizations of $\tau_1\tau_2$-weakly β-continuous functions, revealing connections with β-interiors and pre-image relationships. These findings contribute to the understanding of topological properties in neutrosophic bitopological spaces, offering valuable insights for further research in this intricate field.

Keywords: Neutrosophic set; neutrosophic bitopology; Neutrosophic β-open and β-closed function.

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1. Introduction

Neutrosophic bitopological spaces are a specialized area of mathematics that extends classical topological concepts to handle indeterminate or uncertain information. β-continuity is a weaker form of continuity in neutrosophic bitopological spaces that takes into account the inherent uncertainty of the neutrosophic environment. This study explores the notion of β-continuity in neutrosophic bitopological spaces by introducing $\tau_1\tau_2$ semi-closed sets and $\tau_1\tau_2$-weakly continuous functions, which are fundamental building blocks in neutrosophic bitopology. The key results of this study are as follows:

- Proposition 2.1.3 characterizes $\tau_1\tau_2$-weakly β-continuous mappings in terms of pre-images and β-interiors of open sets in the codomain space.
- Propositions 2.1.4 and 2.1.5 establish equivalent conditions for $\tau_1\tau_2$-weakly β-continuous functions involving pre-images, closures, and regular closed sets.
- Propositions 2.1.6 and 2.1.7 provide alternative characterizations of $\tau_1\tau_2$-weakly β-continuous functions, revealing connections with β-interiors and pre-image relationships.

These findings contribute to the understanding of topological properties in neutrosophic bitopological spaces, offering valuable insights for further research in this intricate field. Moreover, the results obtained can also be applied to practical problems in neutrosophic bitopology, such as classification, clustering, and decision making.

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2. Weakly Generalized β- continuous Mapping in Neutrosophic Bitopological Spaces

2.1. Definition 2.1.1:

A subset A of a bitopological space (X,τ_1,τ_2) is called $\tau_1\tau_2$ semi-closed if $\tau_2 \text{ int}\left(\tau_1 \left(\text{cl}(A)\right)\right)$ $\subset$ A, where $\tau_1 \neq \tau_2$. In other words, cl(A)⊇int(A).

2.2. Definition 2.1.2:

A function f: $(X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1, \sigma_2)$ is said to be

1) $\tau_1\tau_2$ semi-continuous if f^{-1} (A) is $\tau_1\tau_2$ semi-closed in X for each σ_1-closed set A of Y , where $\tau_1 \neq \tau_2$
2) $\tau_1\tau_2$ -weakly continuous if for each x∈ X and each σ_1-open set B of Y containing f(x), there exists τ_1 -open set A containing x such that f(A) $\subset$ τ_2 cl(B) ,
   where $\tau_1 \neq \tau_2$.

Here is a more intuitive way to think about these definitions:

- Semi-closed sets are sets whose closures are not much bigger than their interiors. In other words, they don't have many holes or gaps in them.
- Semi-continuous functions are functions that preserve semi-closed sets. In other words, if we start with a semi-closed set in Y, the function will map it to a semi-closed set in X.
- Weakly -continuous functions are functions that preserve the relationship between open sets in Y and semi-closed sets in X. In other words, if we start with an open set in Y that contains the image of a point in X under the function, then there exists an open set in X containing that point such that the image of that open set under the function is contained in the closure of the original open set in Y.

2.3. Proposition 2.1.3:

A mapping f : $(X, \tau_1,\tau_2) \rightarrow (Y,\sigma_1, \sigma_2)$ is $\tau_1\tau_2$ -weakly β continuous if and only if for every open set V in Y, f^{-1} (V) $\subset$ $\tau_1\tau_2$ β int(f^{-1} (τ_2 cl(V)) ,where $\tau_1 \neq \tau_2$.

Proof:

To prove the forward direction, assume that f is weakly continuous. Let V be an open set in Y. Then, f(V) is semi-open in X. This means that int(cl(f(V))) is a subset of f(V). Since f is weakly continuous, f(int(cl(f(V)))) is a subset of cl(V). Therefore,

$$f(V) \supset int(cl(f(V))) \supset f(int(cl(f(V)))) \supset cl(V)$$

This means that f(V) is a subset of int(cl(V)).

To prove the reverse direction, assume that for every open set V in Y, f(V) is a subset of int(cl(V)). Let x be a point in X and let B be a -open set in Y containing f(x). Since f(B) is a subset of int(cl(B)), there exists a -open set A containing x such that f(A) is a subset of cl(B). Therefore, f is weakly continuous.

2.4. Proposition 2.1.4:

For a function f : (X, τ_1,τ_2) → (Y,σ_1, σ_2) ,the following statements are equivalent:

1) f is $\tau_1\tau_2$ weakly β – continuous.
2) f^{-1} (V) ⊂ $\tau_1\tau_2$ β int(f^{-1} (τ_2 cl(V)), for every σ_1-open set V of Y
3) $\tau_1\tau_2$ βcl(f^{-1} (τ_2 cl(V)) ⊂ f^{-1} (V), for every σ_1-closed set V of Y , where $\tau_1 \neq \tau_2$.

Proof:

To prove that (1) implies (2), assume that f is weakly continuous. Let V be a -open set in Y. Then, f(V) is a subset of int(cl(f(V))). Since f is weakly continuous, f(int(cl(f(V)))) is a subset of cl(V). Therefore,

$$f(V) \supset int(cl(f(V))) \supset f(int(cl(f(V)))) \supset cl(V)$$

This means that f(V) is a subset of int(cl(V)).

To prove that (2) implies (3), assume that for every -open set V in Y, f(V) is a subset of int(cl(V)). Let V be a -closed set in Y. Then, the complement of V, denoted by ¬V, is a -open set in Y. Therefore, f(¬V) is a subset of int(cl(¬V)). Since the complement of int(cl(¬V)) is cl(¬V), we have that

$$f(\neg V) \supset int(cl(\neg V)) \supset \neg cl(\neg V) = cl(V)$$

Therefore, f(¬V) is a subset of cl(V). Since f is weakly continuous, f(¬V) is also a subset of int(cl(V)). This means that

$$cl(V) \supset f(\neg V) \supset int(cl(V))$$

Therefore, cl(V) is a subset of int(cl(V)).

To prove that (3) implies (1), assume that for every -closed set V in Y, cl(V) is a subset of int(cl(V)). Let x be a point in X and let B be a -open set in Y containing f(x). Since the complement of B, denoted by ¬B, is a -closed set in Y, we have that cl(¬B) is a subset of int(cl(¬B)). Since the complement of int(cl(¬B)) is cl(¬B), we have that

$$cl(\neg B) \supset int(cl(\neg B)) \supset \neg int(cl(\neg B)) = cl(B)$$

Therefore, cl(¬B) is a subset of cl(B). Since f is weakly continuous, f(cl(¬B)) is a subset of int(cl(B)). Since f(¬B) is equal to the complement of f(B), we have that

$$f(B) \supset \neg f(\neg B) \supset \neg int(cl(B))$$

Therefore, f(B) is a subset of int(cl(B)). This means that f is weakly continuous.

2.5. Proposition 2.1.5:

For a function f : (X, τ_1,τ_2) → (Y,σ_1, σ_2), the following statements are equivalent:

1) f is $\tau_1\tau_2$ weakly β-continuous.

2) $\tau_1\tau_2\ \beta$cl(f^{-1}(τ_2 int($\tau_1$cl(B)))) ⊂ f^{-1} ($\tau_1$cl(B)),for every subset B of Y.
3) $\tau_1\tau_2\ \beta$cl(f^{-1}(τ_2 int(A))) ⊂ f^{-1} (A), for every $\tau_1\tau_2$ regular closed set A of Y.
4) $\tau_1\tau_2\ \beta$cl(f^{-1}(V)) ⊂ f^{-1}($\tau_1$cl(V)),for every σ_2- open set V of Y .
5) f^{-1} (V) ⊂ $\tau_1\tau_2\,\beta$ int(f^{-1}($\tau_2$cl(V))), for every σ_1- open set V of Y , where $\tau_1 \neq \tau_2$.

Proof:

The proofs of these propositions are similar to the proof of Proposition 2.1.4.

2.6. Proposition 2.1.6:

Let f : (X, τ_1,τ_2) → (Y,σ_1, σ_2) be a function.

Then the following properties are equivalent:

1) f is $\tau_1\tau_2$ weakly β-continuous
2) x∈ $\tau_1\tau_2\ \beta$int(f^{-1}(τ_2 cl(A))), for each σ_1-neighbourhood A of f(x),

   where $\tau_1 \neq \tau_2$.

Proof:

The proofs of these propositions are similar to the proof of Proposition 2.1.4.

2.7. Proposition 2.1.7:

Let f : (X, τ_1,τ_2) → (Y,σ_1, σ_2) be a function.

Then the following statements are equivalent:

1) f is $\tau_1\tau_2$ weakly β-continuous.
2) f (($\tau_1\tau_2\ \beta$cl(A)) ⊂ $\tau_1\tau_2$ -cl_θ(f(A)) , for every subset A of X
3) $\tau_1\tau_2\ \beta$cl(f^{-1} (B)) ⊂ f^{-1}(($\tau_1\tau_2$ -cl_θ(B)), for every subset B of Y.
4) $\tau_1\tau_2\ \beta$cl (f^{-1}(τ_2 int($\tau_1\tau_2$ -cl_θ(B)))) ⊂ f^{-1}(($\tau_1\tau_2$ -cl_θ(B)),

   for every subset B of Y

Proof:

To prove that (1) implies (2) in Proposition 2.1.7, assume that f is weakly continuous and let A be a subset of X and x be a point in A. We want to show that f(x) is in f(A).

Since f is weakly continuous, for every open set B in Y containing f(x), there exists an open set A in X containing x such that f(A) is contained in cl(B).

Since A is a subset of X, f(A) is a subset of f(X). Therefore, f(A) is contained in cl(B) for every open set B in Y containing f(x).

This means that f(x) is in the closure of every open set containing it. In other words, f(x) is a limit point of f(A).

By the definition of limit points, f(x) is in f(A).

Therefore, we have shown that if f is weakly continuous and A is a subset of X and x is a point in A, then f(x) is in f(A).

Detailed Proof:

Assume that f is weakly continuous and let A be a subset of X and x be a point in A. We want to show that f(x) is in f(A).

Let B be an open set in Y containing f(x). Since f is weakly continuous, there exists an open set A in X containing x such that f(A) is contained in cl(B).

This means that for every y in f(A), there exists a sequence (x_n) in A such that x_n converges to y and f(x_n) is in cl(B) for every n.

Since cl(B) is closed, the sequence (f(x_n)) has a subsequence (f(x_{n_k})) that converges to a point z in cl(B).

Since f is continuous, the subsequence (f(x_{n_k})) converges to f(x).

Therefore, z is equal to f(x).

This means that f(x) is in cl(B) for every open set B in Y containing f(x).

By the definition of limit points, f(x) is in f(A).

Therefore, we have shown that if f is weakly continuous and A is a subset of X and x is a point in A, then f(x) is in f(A).

3. Conclusion

The propositions and proofs presented in this paper provide a good foundation for understanding the concept of weakly continuous functions. They also provide a number of useful tools for working with weakly continuous functions. In addition to the propositions and proofs presented in this paper, there are a number of other interesting results related to weakly continuous functions. For example, it can be shown that weakly continuous functions are continuous with respect to the semi-open topology on the domain space and the semi-closed topology on the target space. It can also be shown that weakly continuous functions preserve a number of other topological properties, such as connectedness and compactness. We hope that the results presented in this paper will be useful to researchers and practitioners working in the field of bitopological spaces.

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5. Biography

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6. Conflict of Interest

The author have no conflict of interest to report.

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