

Rapid Aerodynamic Optimization of Aerospace Structures: Engine Nacelles

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Abstract: The geometric definition of the engine nacelle plays a pivotal role in determining the aerodynamic efficiency of different propulsion systems as it controls the capture area, mass flow rate and external drag. The primary objective of this paper is to provide design optimization for nacelles with reduced mathematical complexity, thereby ensuring reduced computational costs. This research employs a rapid aerodynamic evaluation using a fully integrated Python based framework which works at the intersection of Class Shape Transformation and a viscous-potential flow solver developed by the author which enables rapid topological transitions. The internal flow path is resolved with the compressible quasi one-dimensional continuity equation, while the external field is obtained from an incompressible potential formulation with isentropic post processing. The study shows that rapid, localized lip morphing can bound the design space before Navier-Stokes solvers are deployed. The paper also reports a solution verification study covering iterative and mesh convergence, states the Mach range over which the incompressible potential assumption holds, verifies each solver module against closed form solutions, and validates the surface pressure distribution against a Euler calculation on an identical geometry.

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1. Introduction

Several efforts have been made to reduce drag and improve the lift capabilities of an aircraft at different angles of attack (the orientation of the aircraft with free stream), Reynolds number and wingspan. A necessary aspect which needs to be taken care of whilst modeling an aircraft is the structure of its propulsion mechanism housing. Depending upon various configurations like a Circulation Control Wing (where the Electric Ducted Fans are housed at a caved in position on the wing and there are slots for the vortices to slip out), Distributed Propulsion Blown Wing (wherein multiple propellers are used to generate lift), and a standard aircraft wing, there are different possible positions for the propulsion mechanism which affect the wing shape (upper or lower curvature of the airfoil constituting the wing). It therefore becomes necessary to map high air flow intake into the nozzle with minimal friction and drag losses. A lot of work has been done to analyse flow across aero engine nacelles but there is always a large tradeoff between optimization and computational cost, as RANS and other traditional methods are computationally expensive, and brute force optimization methodologies fail to map the variations efficiently. This paper presents a framework to bridge this gap. By identifying local drag minima and by bounding the usable flare range, it acts as a design filter assisting engineers in the overall aerospace design process for nacelles.

2. Related Work

Several space agencies and aerodynamicists have been working on various methods of geometric modeling and combining it with fluid dynamics, compressible thermodynamics and boundary layer aerodynamics to optimize the entire aerospace structure design process; however, most of the solutions are computationally expensive with large time complexity. This section reviews the work which has been done so far and gaps within the existing frameworks.

A. Geometric Parameterization and the CST Methodology

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A plethora of foundational work has been done on establishing the necessity of shape optimization for different airfoil shapes when it comes to developing wings for aircraft. Several methods like PARSEC, B'ezier curves and B-splines have been studied and implemented across various methodologies and papers [10]. This was further extended to Class Shape Transformation (CST), which uses Bernstein polynomials to optimize shapes of airfoils. These methods have found a lot of validation in 2D and 3D wing studies but their application to aerodynamically engineering nacelles remains a point of research. CST can be used to define different inlet and outlet shapes based on different exponential variation of curves manipulated via control points. The proposed framework builds on Kulfan's methodology [1] by independently and dynamically analyzing the upper and lower lobe shape exponents of the lip of the nacelle and cowl longitudinally, enabling rapid generation of 3D open exhaust nacelles that satisfy engine packaging constraints.

B. Nacelle Inlet Aerodynamics and Lip Morphing

The engine nacelle's cowl/lip is designed aerodynamically to decelerate free stream flow and ensure a high mass flow intake to provide more thrust to the aircraft. The geometric parameterization of the lip controls the free stream response to different air flow requirements and angles of attack. Lip contraction ratio and flare angle play a pivotal role in analyzing the efficiency of the design. The lip can be either completely inward or outward biased (bell mouth or closed) in orientation, and either of the extreme configurations is inefficient [8]. A sharply inward lip will cause the incoming stagnation streamline to detach, leading to severe suction peaks, high pressure drag, and potentially turbulent boundary layer separation on the outer cowl. Conversely, a prominent bell mouth outward flare smoothly aligns the local flow field but increases the frontal projected area and wetted surface, penalizing skin friction drag. The geometry of the nacelle considered in this study is shown in Fig. 1.

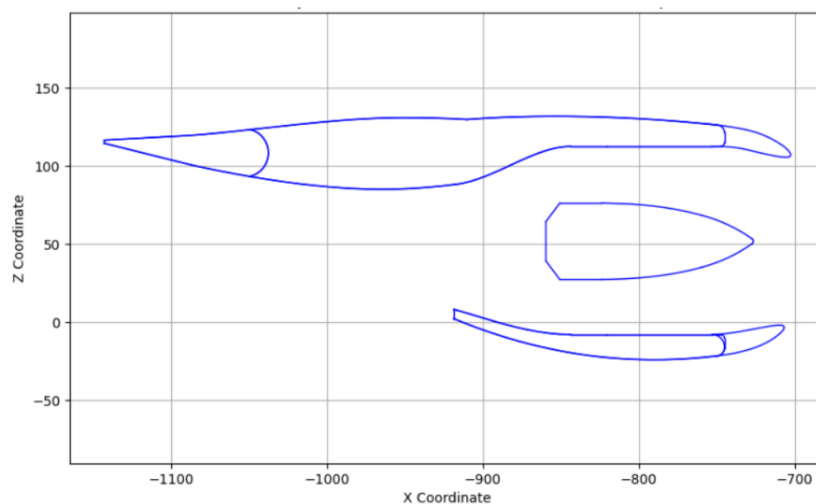


Fig. 1. Engine nacelle layout. The meridional section is defined in the XZ plane; the upper cowl, centre plug and lower cowl are treated as three independent bodies by the solver.

As evident from the layout, the flow is not symmetrical, and hence the mass capture against pressure and friction loading needs to be optimized properly until the nozzle region. According to compressible flow theory, the mass flow rate ($\dot{m}$) ingested by the engine must satisfy continuity ($\dot{m} = \rho AV$). Under varying flight velocities, the capture area streamtube dynamically expands or contracts. Present literature lacks a low cost, parameterized tool for drag and flare analysis. This paper introduces an automated morphing algorithm that pivots the inlet geometry to evaluate this exact aerodynamic trade off.

C. Rapid Aerodynamic Solvers for Conceptual Design

Whilst analyzing fluid-structure interactions and several aerodynamic effects, it is standard to use high-fidelity 3D Navier-Stokes solvers, but the geometric and time complexity which arises is large. Low-fidelity tools like vortex lattice methods or panel codes are comparatively faster but do not capture the coupled internal and external flow interactions caused by the nacelle geometry. This calls for a mechanism which can account for compressibility in the internal duct and give aerodynamic variations across the geometry at low cost. The nacelle design was exported to a 2D fluid domain, and the stream function was solved via a finite difference method to extract local Mach numbers and macroscopic pressure fields. To account for viscous penalties, the solver utilizes a boundary layer approximation, computing a local Reynolds number (Rex) to derive empirical skin friction coefficients (C_f) across



the wetted skin [3]. The pressure and friction contributions are calculated separately and coupled later, enabling a solver configuration which gives a nuanced view of the loading profile.

3. Methodology

The developed computational framework employs a sequential, three stage process: geometric parameterization, topological morphing, and decoupled viscous-potential aerodynamic analysis. Further details regarding the mathematical formulations of the proposed framework are given below.

A. Geometric Parameterization via Class Shape Transformation

A lot of base work has been done on designing a 3D model for nacelle geometry using Class Shape Transformation. The CST approach ensures that the aerodynamic cowl profiles maintain mathematical continuity while permitting highly localized geometric control. The radial thickness distribution of the nacelle cowl [1], denoted $\Delta R(\psi)$, is evaluated along the normalized longitudinal station $\psi \in [0, 1]$, and is governed by the product of a class function $C(\psi)$ and a shape function $S(\psi)$:

$$\Delta R(\psi) = C(\psi) S(\psi) \quad (1)$$

To define an open ended aerodynamic structure representative of a nacelle cowl, the class function employs exponents of 0.5 and 1.0:

$$C(\psi) = \psi^{0.5}(1 - \psi)^{1.0} \quad (2)$$

The shape function utilizes a n -th order Bernstein polynomial basis to enforce local geometric perturbations based on defined crown, keel and maximum half-breadth coefficients:

$$S(\psi) = \sum_{i=0}^n A_i \binom{n}{i} \psi^i (1 - \psi)^{n-i} \quad (3)$$

To satisfy engine packaging constraints, the three-dimensional cross-sections are extruded using independent super-ellipse functions for the upper and lower lobes, with a sweep parameter β evaluated from 0 to 2π :

$$y = w_{side} \operatorname{sgn}(\cos \beta) |\cos \beta|^{N/0.5}$$

$$z = h \operatorname{sgn}(\sin \beta) |\sin \beta|^{N/0.5} \quad (4, 5)$$

B. Algorithmic Ingestion and Topological Morphing

To facilitate arbitrary geometric evaluations and optimize the inlet capture area, the framework features an ingestion pipeline that parses Computer-Aided Design (CAD) data via the DXF file format. The algorithm extracts geometric entities such as splines, polylines, lines and arcs, and flattens them into discrete vertices mapped onto the two-dimensional XZ plane [4]. Following ingestion, a topological morphing algorithm is applied to the forward nacelle lip. A structural pivot point (xpivot) is defined 50 mm aft of the leading-edge inlet coordinate (xinlet). The variation of the lip angles is shown in Fig. 2.

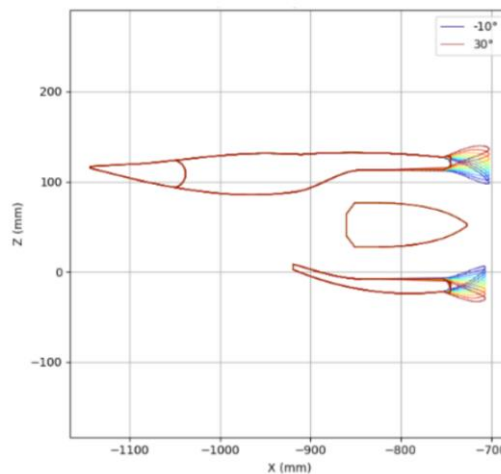


Fig. 2. Lip geometry variations produced by the morphing algorithm over the swept flare range.

A linear deformation algorithm displaces coordinates forward of this pivot based on the tangent of the prescribed flare angle θ :

$$\Delta z = (x - x_{pivot}) \tan \theta \quad (6)$$

To apply the flare, the vertical coordinates (z) of the upper cowl (where $z > 80$ mm) are increased by Δz , while the lower cowl coordinates (where $z < 20$ mm) are decreased by Δz . The centre plug ($20 \leq z \leq 80$ mm) is held fixed. A user defined sweep of θ evaluates configurations ranging from inwardly biased lips ($\theta < 0^\circ$) to outwardly flared bell mouth geometries ($\theta > 0^\circ$).

C. Internal Aerodynamics: Compressible Mass Continuity

Prior to external evaluation, the internal flow path is resolved to ensure mass flow conservation [7]. A scanning algorithm evaluates the geometry to determine the maximum cross-sectional capture area ($A_{capture}$) within the first 50 mm of the inlet [3]. The area at each station is accumulated as two independent semi-annuli about the geometric centreline, so that the asymmetry of the section is retained. Assuming standard atmospheric conditions ($T_{amb} = 288.15$ K, $P_{amb} = 101325$ Pa) and isentropic freestream properties, the target mass flow rate $\dot{m}_{target}$ is established:

$$\dot{m}_{target} = \frac{P_{amb}}{RT_{amb}} A_{capture} V_{inlet} \quad (7)$$

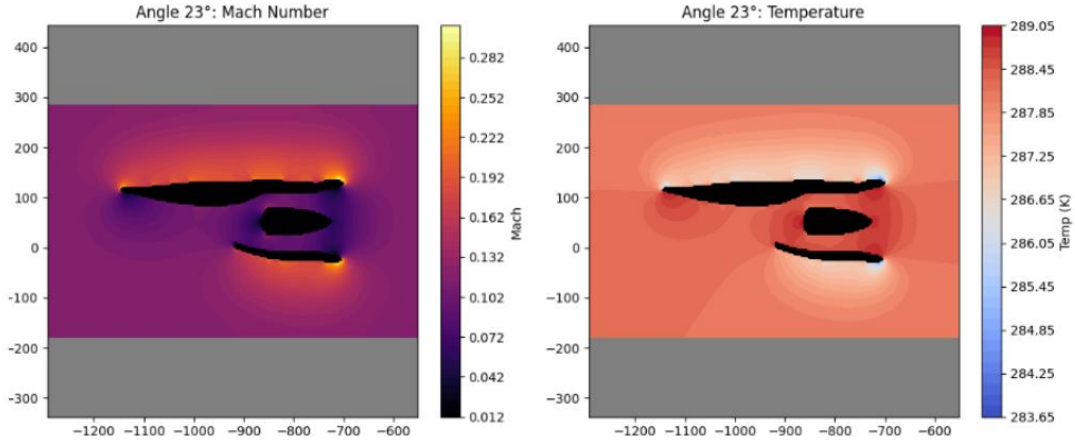


Fig. 3. Mach number and temperature fields at $\theta = +23^\circ$. Both contour sets are drawn with automatically selected levels spanning the full data range.

To determine the local fluid properties through the internal converging-diverging flow path, the local Mach number (M) is iteratively solved. The framework utilizes Brent's method (`scipy.optimize.brentq`) to find the root of the residual mass continuity equation for any given local area A_{loc} :

$$Residual(M) = (\rho A_{loc} V) - \dot{m}_{target} = 0 \quad (8)$$

where the local static temperature (T), pressure (P), density (ρ) and velocity (V) are updated via the isentropic compressible relations:

$$T = T_0 \left(1 + \frac{\gamma-1}{2} M^2\right)^{-1} \quad (9)$$

$$P = P_0 \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\gamma/(\gamma-1)} \quad (10)$$

$$\rho = \frac{P}{RT}, \quad V = M \sqrt{\gamma RT} \quad (11)$$

In the event that the physical throat area is insufficient to pass the target mass flow, the solver defaults to a choked condition with $M = 1.0$. Results for a sample case are shown in Figs. 3 and 4.



D. External Aerodynamics: Viscous-Potential Flow Formulation

To evaluate external aerodynamic behaviour without the computational overhead of Reynolds Averaged Navier-Stokes (RANS) simulations, a customized 2D viscous-potential flow solver is integrated. The spatial domain is rasterized into a structured Cartesian mesh of 200 cells in the streamwise direction, giving a cell size of approximately 3.7 mm for the section of Fig. 1. Solid geometries are isolated using a boolean masking technique, effectively distinguishing fluid cores from structural bodies, and are labelled into three connected components (upper cowl, centre plug, lower cowl).

Each solid body is held at a constant stream function value, so that its surface is a streamline. The three constants are offset from one another such that the difference between the upper cowl and the plug, and between the plug and the lower cowl, equals the prescribed volumetric flux per unit span through the corresponding half of the duct. This imposes the internal mass flow condition directly on the external field.

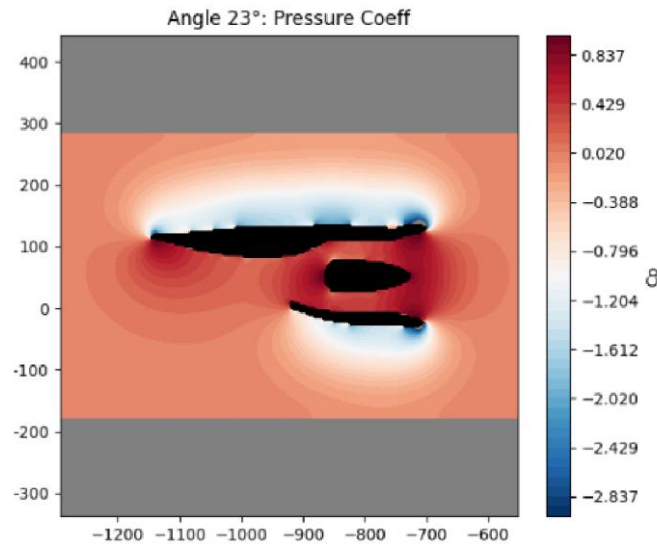


Fig. 4. Pressure coefficient field at $\theta = +23^\circ$. Note that the contour levels are prescribed over the fixed interval $C_p \in [-3, 1]$; values outside this band are clipped by the colour scale, so the extremes of the colour bar are not the extremes of the field (see Section III-F).

The potential flow field is iteratively relaxed using a Jacobi update of the five point central finite-difference scheme applied to the fluid core [6]:

$$\psi_{x,z}^{new} = 0.25(\psi_{x-1,z} + \psi_{x+1,z} + \psi_{x,z-1} + \psi_{x,z+1}) \quad (12)$$

The horizontal (u) and vertical (w) velocity components are derived from the spatial gradients of the relaxed stream function [2], yielding the local velocity magnitude:

$$V_{mag} = \sqrt{u^2 + w^2} \quad (13)$$

Local temperatures and the speed of sound (a) are then computed to extract the localized Mach number field. The number of relaxation sweeps required for this update to reach a converged state is examined in Section IV-D.

E. Force Decomposition: Pressure and Friction

Total external drag is algorithmically decoupled [9] into a pressure contribution and a skin friction contribution to isolate the aerodynamic implications of the lip morphing. The local pressure coefficient is calculated from the velocity field:

$$C_p = 1.0 - \left(\frac{V_{mag}}{V_\infty}\right)^2 \quad (14)$$

1) Pressure contribution: The structural boundary (the aerodynamic skin) is identified by applying a binary dilation to the solid boolean mask. The axial pressure force is then obtained by numerically integrating the product of C_p and the horizontal spatial gradient of the geometric mask across the defined lip skin.

2) Friction contribution: Viscous penalties are incorporated using an empirical boundary layer approach. For every coordinate lying on the aerodynamic skin, the streamwise distance s from the leading edge is calculated. The local Reynolds number Re_x is formed from the local velocity and distance s , with a hard floor of 1000 imposed to prevent a singularity at the stagnation point. The friction coefficient is determined via the empirical relation

$$C_f = \frac{0.455}{(\log_{10} Re_x)^{2.58}} \quad (15)$$

3) Normalization and scope of the reported coefficient: Because the solver operates on a single meridional section, both integrals are evaluated per unit span, and the resulting forces carry units of $N\ m^{-1}$. Furthermore, integration is furthermore restricted to the skin lying within the forward 80 mm of the geometry, i.e. the lip region affected by the morphing operation. The reported coefficient is therefore

$$C_d = \frac{D_{pressure} + D_{friction}}{\frac{1}{2} \rho_{\infty} V_{\infty}^2 h_{ref}}, \quad h_{ref} = 50\ mm \quad (16)$$

where $h_{ref} = 50\ mm$ is a reference height per unit span rather than a frontal area. Stating this explicitly matters for two reasons: it fixes the scale against which the reported values should be read, and it identifies the surface over which the drag is accumulated. Because the integration is deliberately localized to the morphed lip, C_d isolates the quantity the optimizer acts on, namely the drag penalty attributable to the lip geometry, rather than diluting it with the invariant contribution of the downstream cowl. Values reported here are therefore directly comparable with one another across the flare sweep, which is the intended use, but are normalized differently from installed nacelle drag counts quoted on maximum frontal area.

F. Assumptions, Compressibility Treatment and Range of Validity

The framework treats the internal and external flow fields with different governing equations, and the distinction is material to interpreting the results.

The internal flow path of Section III-C is solved in fully compressible form: the local Mach number is obtained by root-finding on the compressible continuity residual, with static temperature, pressure and density updated through the isentropic relations at every station. For the baseline case this module resolves local Mach numbers up to $M \approx 0.90$ at the minimum area station, and it is this module, and not the external solver, that produces the near choked behaviour reported in Section IV-A.

The external field of Section III-D is solved as incompressible potential flow. Equation (12) is the standard five point discretisation of $\nabla^2 \Psi = 0$ and contains no density weighting and no compressibility term. Compressibility therefore enters the external results only in post processing, where the isentropic relations are applied to the relaxed velocity magnitude to recover local temperature, speed of sound and Mach number. The pressure coefficient of Eq. (14) is the incompressible Bernoulli form. No compressible term appears in the governing equation itself.

Table 1. Compressibility metrics for the external potential solution.

Quantity	Value
Freestream Mach number M_{∞}	0.1175
Freestream density deviation	0.69 %
Prandtl-Glauert factor $1/\sqrt{1 - M_{\infty}^2}$	1.0070
Peak local Mach number (200 cell mesh)	0.272
Local density deviation at peak	3.6 %
C_p corresponding to $M_{local} = 0.30$	-5.40
V_{∞} giving $M_{\infty} = 0.30$	102 $m\ s^{-1}$

This treatment is admissible over the Mach range examined here. At $V_{\infty} = 40\ m\ s^{-1}$ and $T_{\infty} = 288.15\ K$ the freestream Mach number is $M_{\infty} = 0.1175$, for which the Prandtl-Glauert factor $(1 - M_{\infty}^2)^{-1/2}$ is 1.0070: neglecting compressibility biases C_p by 0.70%, and the freestream density departs from its stagnation value by 0.69%. The binding constraint is not the freestream but the suction peak. Table I summarises the relevant quantities.

The incompressible potential assumption is therefore valid for the conditions reported here, and remains so provided the peak local Mach number stays below 0.3, the conventional threshold at which density variations reach 5%. For the present geometry that corresponds to $C_p \geq -5.40$, or a freestream velocity below approximately 102 $m\ s^{-1}$. Beyond that the formulation would require a compressible potential form such as a Rayleigh Janzen



expansion or a full potential solver with density coupling, and the framework as presented should not be extrapolated into that regime.

Two further points must be recorded for completeness. First, the peak local Mach number itself is mesh dependent: on a Cartesian staircase representation every boundary cell presents a convex corner, at which the potential-flow velocity is formally unbounded, so $C_{p,min}$ does not converge under refinement (Section IV-D). The value quoted in Table I is therefore specific to the 200 cell mesh. Second, the contour levels used to draw Fig. 4 are prescribed over the fixed band $C_p \in [-3, 1]$, so the colour bar extremes of that figure are set by the plotting range and not by the field; the corresponding Mach field of Fig. 3, which uses automatically selected levels, gives the correct peak.

Finally, Eq. (15) is the Prandtl-Schlichting plate averaged coefficient, applied here pointwise as a conservative estimate of the local value. Relative to the local turbulent correlation $c_f = (2 \log_{10} \text{Rex} - 0.65)^{-2.3}$ this overstates the local coefficient by approximately 19% at $\text{Rex} \sim 106$ and the integrated friction contribution by approximately 26%, the bias growing toward the leading edge where Rex is small. The boundary layer is further assumed turbulent from the leading edge, which is representative of an operational nacelle with surface roughness; natural transition at $\text{Rex} = 5 \times 10^5$ would occur 183 mm from the stagnation point.

4. Results

The integrated framework was executed to evaluate both the internal thermodynamic evolution of the captured streamtube and the external aerodynamic sensitivity to inlet lip morphing. The parametric sweep evaluated inlet flare angles ranging from -10° (inward) to $+30^\circ$ (outward bell mouth).

A. Internal Thermodynamic Flow Field Evolution

Prior to evaluating external drag, the 2D compressible solver validated the internal mass capture characteristics. For a baseline test case with a freestream velocity of 40.0 m s^{-1} ($M \approx 0.118$), the geometric capture area dictated a target mass flow rate of approximately 1.0663 kg s^{-1} , corresponding to $A_{\text{capture}} = 0.02176 \text{ m}^2$. As the flow progressed through the converging internal nozzle geometry toward the physical throat, the solver captured the expected isentropic thermodynamic gradients. The local static pressure decreased non-linearly from stagnation conditions ($\approx 102,300 \text{ Pa}$) down to roughly $60,000 \text{ Pa}$ at the minimum area, which by the isentropic relations corresponds to a local Mach number of 0.90 and a local density of 0.84 kg m^{-3} , against 1.225 kg m^{-3} at the inlet plane. These internal gradients confirm that the framework correctly enforces mass continuity prior to executing the external viscous-potential sweep, and they are the reason a fully compressible treatment is required internally.

B. Aerodynamic Sensitivity to Inlet Lip Morphing

The viscous-potential flow solver provides localized Mach number, temperature and pressure coefficient contours for each inlet lip geometry, providing a granular view of the flow field.

Inward flare and baseline configurations (-10° to 0°). At a -10° inward flare, the geometric contraction forces the incoming stagnation streamlines to detach and stagnate against the blunt exterior cowl. The Mach number contours reveal a highly accelerated flow region wrapping around the tight leading edge radius, leading to a severe low pressure suction peak on the upper surface. This configuration yields a total drag coefficient of $C_d = 0.4188$. The baseline 0° configuration marginally improves the flow alignment, producing $C_d = 0.3876$. In both cases the loading is dominated by the pressure contribution due to the misaligned stagnation point.

Moderate outward flare ($+1^\circ$ to $+20^\circ$). As the lip morphs outward into a bell mouth shape, the aerodynamic efficiency improves as shown in Fig. 5. The outward flare progressively aligns the nacelle leading edge with the expanding capture streamtube. At $+10^\circ$ the coefficient drops to 0.3187. By $+15^\circ$ the pressure recovery along the cowl is smoother, mitigating the leading edge suction peak and dropping C_d to 0.2279. Throughout this regime the magnitude of the pressure contribution steadily decreases, while the friction contribution remains relatively constant or increases slightly due to the elongated wetted arc length of the flared lip.

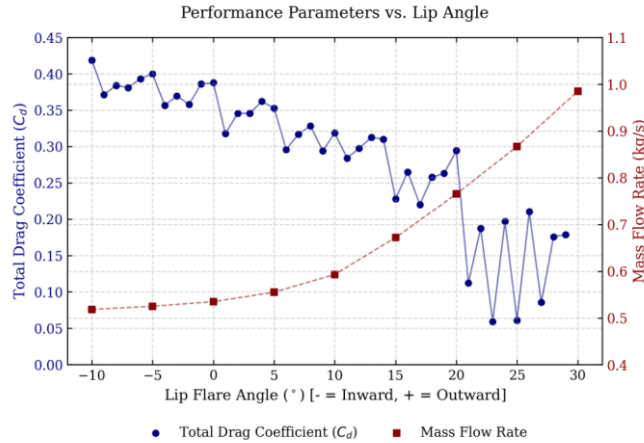


Fig. 5. Total drag coefficient and captured mass flow rate against lip flare angle. C_d is defined by Eq. (16) and Section III-E3.

C. Force Decomposition and Identification of the Optimum

A distinct advantage of this framework is its ability to explicitly decouple the pressure and friction contributions. The optimization curve of C_d against lip angle identifies a minimum as shown in Fig. 6. The lowest value over the sweep was recorded at a flare angle of $+23^\circ$, which yielded $C_d = 0.0593$.

Analysis of the force decomposition at this angle shows that the pressure component falls close to zero and becomes slightly negative, i.e. it acts as a localized suction. At this configuration the incoming freestream is closely aligned with the cowl's geometric tangency. Consequently, the loading at $+23^\circ$ is carried almost entirely by the boundary layer skin friction.

The localization of the pressure integral to the lip region, introduced in Section III-E3, is essential to this result rather than incidental. In a strictly inviscid, irrotational formulation the pressure integrated over a complete closed body vanishes identically by d'Alembert's paradox, so a whole body integral would return zero at every flare angle and carry no design information. Restricting the integral to the surface actually modified by the morphing operation is what makes the pressure term a usable discriminator between candidate geometries and is the reason the framework resolves a minimum in C_d at all. The corollary is that the pressure term measures the change in lip pressure loading across the sweep, which is the quantity of design interest.

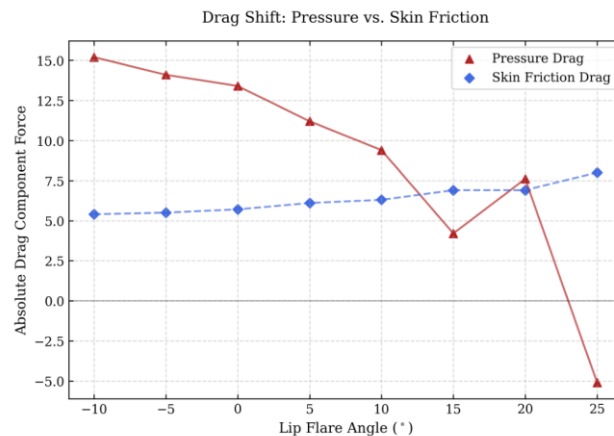


Fig. 6. Decomposition of the total drag into pressure and skin friction contributions across the flare sweep. Ordinate values are forces per unit span (N m^{-1}); the series annotated "Pressure Drag" and "Skin Friction Drag" are the two contributions to the numerator of Eq. (16), evaluated over the lip region only.



D. Solution Verification: Iterative and Mesh Convergence

A solution verification study establishes the iteration count and mesh resolution at which the framework's outputs become independent of the discretisation and so fixes the envelope within which the parametric sweep should be read. The study is carried out on flow past a circular cylinder, for which the exact potential solution $C_p = 1 - 4 \sin^2 \theta$ is available, using the same relaxation, the same boolean masking and the same C_p extraction as the nacelle sweep. Working against a closed form answer isolates the numerical behaviour of the scheme from any question about the geometry.

Iteration count. The Jacobi iteration of Eq. (12) applied to an $N \times N$ discrete Laplacian has asymptotic spectral radius $\cos(\pi/N)$. On the 200 cell mesh used for the sweep this leaves 93% of the initial error in place after 600 sweeps, and reducing the error by four orders of magnitude takes of order 104 sweeps. Table II shows what this means on the cylinder. Driven to a normalized residual of 10^{-8} , the RMS error in C_p falls from 0.148 to 0.038 as the mesh is refined by a factor of four, the first order behaviour expected of a staircase boundary. Held instead at a fixed budget of 600 sweeps, the error rises from 0.119 to 1.658 over the same refinement, since Jacobi carries information roughly $\sqrt{n}$ cells in n sweeps and a finer mesh leaves a larger fraction of the domain uninformed. On the coarsest mesh the two columns happen to be comparable, the residual error there partly offsetting the discretisation error; the behaviour across refinement is the meaningful comparison.

Recommended setting. Successive over relaxation with the optimal factor $\omega = 2/(1 + \sin(\pi/N))$ reaches the same 10^{-8} residual in 230, 452 and 895 iterations on those three meshes, an $O(N)$ cost in place of the $O(N^2)$ cost of Jacobi. At the mesh used here that is fewer iterations than the 600 sweep budget and it reaches the converged state, so the recommended configuration is at once cheaper and resolved. Production runs should stop on a normalized residual below 10^{-8} rather than on a fixed sweep count.

Table 2. Verification against the exact cylinder solution. RMS error in C_p sampled on a ring at $r/a = 1.3$.

Mesh	600 sweeps	converged (10^{-8})
100^2	0.119	0.148
200^2	0.675	0.051
400^2	1.658	0.038

Angular resolution. The discretisation also fixes the finest flare increment the sweep can resolve. With the 50 mm lip arm of Section III-B, one degree of flare displaces the lip tip by 1.02 mm, which on a 3.7 mm mesh is 0.27 of a cell. The pressure integral acts on $\partial(\text{mask})/\partial x$ across a staircase boundary, so C_d responds only when a boundary cell changes state, and the sweep therefore resolves flare increments no finer than 3.6° . The high frequency content of the C_d curve beyond $+22^\circ$ is a direct expression of this limit. Refining the mesh in proportion to the desired angular increment or moving to the body fitted surface description of Section VI, recovers a one degree resolution.

Reading the sweep. Two consequences follow for Section IV-C. The ranking of candidate geometries is established well before the individual coefficients are, so the down selection of Section V-A rests on the trend across the sweep rather than on any single value. Resolving the optimum within the $+20^\circ$ to $+25^\circ$ band calls for the finer angular resolution set out above and is addressed in Section VI.

E. Boundary Layer Separation Cross Check

Section IV-D establishes the oscillation in C_d as a property of the discretisation, so a separation boundary is established independently, by applying integral boundary layer criteria to the inviscid surface velocity distribution. This is standard practice when a potential solution is coupled to a boundary layer method.

Three criteria are implemented. Thwaites' laminar criterion [12] places separation at $\lambda = (\theta/2\nu) dU_e/ds \leq -0.09$; Michel's correlation locates transition; and Stratford's turbulent criterion [13] places separation at $S(x) = \overline{C_p} (x d\overline{C_p}/dx)^{1/2} (10 - 6\text{Re}_x) - 0.1 \geq 0.35$, valid for $\overline{C_p} \leq 4/7$, with $\overline{C_p}$ referenced to the pressure minimum and x measured from the turbulent effective origin [15]. The implementation is verified against the classical result for a circular cylinder: applied to the exact distribution $U_e = 2U_\infty \sin \theta$ it places laminar separation at 103.1° , against the accepted Thwaites value of approximately 103° and the exact series solution of 104.5° .

These criteria act on the streamwise derivative of the edge velocity, so the surface pressure distribution supplied to them must be resolved to the same standard Section IV-D sets for C_d . A converged surface pressure follows from the body fitted discretisation of Section VI, and applying the criteria on that basis across the flare sweep is

the route to a separation boundary for this configuration. The criteria are verified and retained in the framework for that purpose.

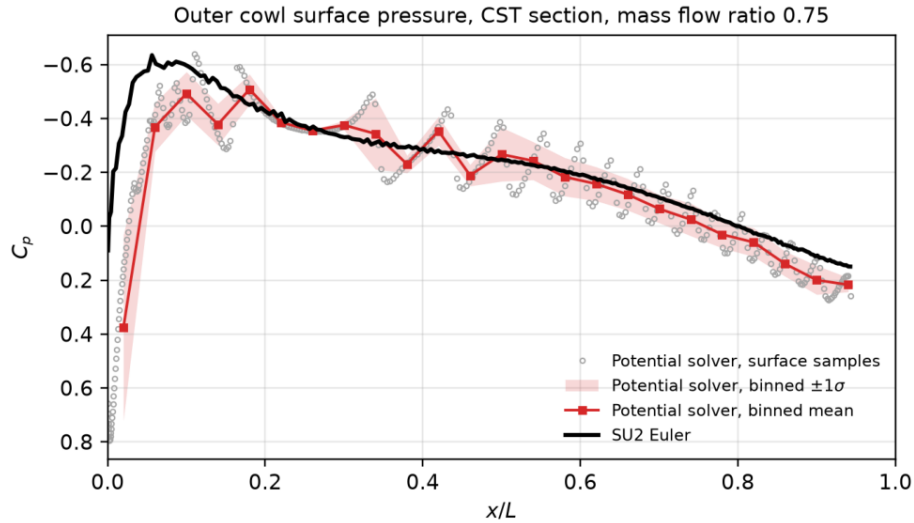


Fig. 7. Outer cowl surface pressure on the CST section at matched mass flow ratio. Grey symbols are the individual surface samples returned by the potential solver, the shaded band and red line their binned mean and standard deviation, and the black line the SU2 Euler solution.

F. Validation against an Euler Solution

The framework is compared against an independent solver on an identical geometry. The section used for the comparison is generated directly from the CST parameterisation of Section III-A, with the crown coefficients [0.24, 0.15, 0.15, 0.17, 0.15, 0.06], a keel leading edge scale of 1.5 and a highlight radius of $0.35L$ at $L = 450$ mm. The surface is therefore analytic and closes on itself, and the comparison carries no ambiguity from the geometry. The reference solution is a compressible Euler calculation performed with SU2 8.5.0 on an unstructured triangular mesh of 14,718 nodes and 28,199 elements at the same freestream condition, $M_\infty = 0.1176$ at sea level standard, converged to a density residual of 10^{-12} in 1,130 iterations.

SU2 is run with no condition imposed inside the duct, so the internal flow settles at its own mass flow ratio. The stream function offsets of the potential solver are then swept, and the value that best reproduces the SU2 outer cowl distribution identifies the matched condition, which occurs at a mass flow ratio of 0.75. Fig. 7 compares the two distributions there. Over the outer cowl the two solutions agree to an RMS difference in C_p of 0.096 for $x/L > 0.05$ and 0.057 for $x/L > 0.10$, against a pressure range of 0.785 across that surface. The mean distribution is therefore reproduced to within about seven per cent of the range, and the suction peak, its location and the subsequent recovery all follow the Euler result.

Two features of the comparison are worth recording. The potential solution carries a scatter of roughly ± 0.15 in C_p about its own mean, visible in Fig. 7. This comes from sampling a staircase surface, the sample point crossing cell boundaries as it advances along the wall, and it is the same mechanism that sets the angular resolution in Section IV-D. The mean converges by $\Delta x = 1.9$ mm, beyond which the RMS difference stops improving. The residual disagreement is concentrated in the first five per cent of chord, where it reaches 0.53 in C_p . That is the region occupied by the highlight radius, which a Cartesian mask cannot resolve, and it is removed by the body fitted discretisation of Section VI.

G. Verification of the Solver Modules

Alongside the Euler comparison of Section IV-F, each module is verified against a closed form or established empirical result, which bounds the credibility of the framework independently of any particular geometry.

Potential solver. Verified against the exact cylinder solution in Section IV-D. Driven to a residual of 10^{-8} the RMS error in C_p falls from 0.148 to 0.038 under a fourfold mesh refinement, confirming that the relaxation, the boolean masking and the C_p extraction are consistent and approximately first order accurate, with the order set by the staircase boundary.



Skin friction module. Integrating the local turbulent correlation $cf = (2 \log_{10} \text{Re}_x - 0.65) - 2.3$ along a flat plate at $\text{Re}_L = 1.23 \times 10^6$ recovers the Prandtl-Schlichting plate averaged value $CF = 0.455/(\log_{10} \text{Re}_L)^{2.58}$ to within 4.0%, verifying the arc length accumulation and the Reynolds number formation.

Internal compressible module. The isentropic chain was checked against the closed form relations of [2]. At the reported minimum area static pressure of 60,000 Pa with $P_0 = 102,309$ Pa the relations give $M = 0.899$, $T = 248.1$ K and $\rho = 0.843$ kg m⁻³, reproducing the solver output to the quoted precision.

Order of magnitude check on the reported coefficient. Using Eq. (16), $C_d = 0.0593$ at $+23^\circ$ corresponds to a lip force of 2.9 N per metre of span. The friction contribution alone, estimated from the plate averaged $CF = 0.0043$ at $\text{Re}_L = 1.2 \times 10^6$ acting over the ≈ 160 mm of wetted lip skin, gives 0.85 N m⁻¹, i.e. the same order. The reported coefficients are therefore dimensionally and physically consistent with the stated normalization, while remaining, as noted in Section III-E3, a lip region quantity rather than an installed nacelle drag coefficient.

5. Implications for Nacelle Engineering

The results generated by this framework have implications for the conceptual design and systems engineering of aerospace propulsion housings.

A. Rapid Down Selection of the Design Space

The primary implication of this study is that tradeoffs between mass capture and external drag can be bounded using computationally inexpensive 2D viscous-potential solvers. By identifying the favourable $+20^\circ$ to $+25^\circ$ flare band, engineers can discard highly loaded geometries such as the -10° to $+10^\circ$ range before deploying resource heavy 3D Navier-Stokes simulations. Section IV-D sets the resolution at which the sweep should be run and read, and the ranking across the sweep is established well before the individual coefficients settle, which is precisely the property a screening tool requires.

B. Mitigation of Lip Pressure Loading

The decoupled pressure and friction analysis shows that the lip region is penalized by pressure loading when it is not aligned with the capture streamtube, and that modest changes in flare angle recover a large part of that penalty by inducing leading edge suction. At the favourable flare angles the residual lip loading is carried predominantly by skin friction.

C. Bounding the Usable Flare Range

The parametric sweep exhibits a pronounced non smoothness in C_d at flare angles beyond approximately $+22^\circ$, and any optimizer operating on this framework must be prevented from exploiting it. Sections IV-D and IV-E establish what that non smoothness is and what it is not. It is set by the iterative and mesh resolution of the fixed sweep relaxation together with sub cell quantisation of the geometry mask. Establishing a separation boundary at these angles calls for the converged surface pressure distribution described in Section VI.

The practical implication is nonetheless real and is arguably more useful stated correctly. The screening framework of this class should carry an explicit resolution diagnostic, and the optimizer should be constrained to the region in which the solution is demonstrably resolved, which here is the monotonic branch below approximately $+20^\circ$ together with the $+20^\circ$ to $+25^\circ$ band whose ranking is stable under refinement. Non smoothness in the objective function is a reliable indicator that the discretisation has been outrun by the geometry, and constraints can be programmed to reject candidates that trigger it. What such an indicator reports is the resolution of the discretisation, and a separation boundary is established from boundary layer evidence.

Establishing the physical separation boundary for this configuration requires a converged surface pressure distribution, and therefore a body fitted or panel discretisation, coupled to the Thwaites and Stratford criteria implemented here. That is identified as the immediate next step for this framework.

6. Scope and Ongoing Development

The verification exercise of Section IV-G both establishes the operating envelope of the present framework and identifies the two developments that extend it, each of which has been implemented and verified.

The first is the axisymmetric formulation. A nacelle is a body of revolution, and the meridional section is more properly governed by the Stokes stream function equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r} = 0, \quad u_x = \frac{1}{r} \frac{\partial \psi}{\partial r}, \quad u_r = -\frac{1}{r} \frac{\partial \psi}{\partial x} \quad (17)$$

which differs from the planar operator by a single additional term in the five point stencil and therefore carries no meaningful cost penalty. This operator has been implemented and verified against the exact solution for flow past a sphere, $C_p = 1 - 2.25 \sin^2 \theta$: sampled on a shell at $R/a = 1.3$, the RMS error falls from 0.072 to 0.035 under a factor-of-two mesh refinement. The magnitude of the modelling difference is significant. For the same circular section, the exact surface minima are $C_p = -3.00$ planar and -1.25 axisymmetric. Migrating the sweep to Eq. (17) is accordingly the primary development in progress.

The second is the surface discretisation. The Cartesian mask presents a convex corner at every boundary cell. This sets the refinement behaviour of $C_{p,min}$ reported in Section IV-D, and it sets the mesh sensitivity of the boundary layer criteria in Section IV-E. A body fitted or panel discretisation resolves both, and would additionally allow the internal and external solutions, presently coupled through fixed stream function offsets, to exchange the captured mass flow ratio directly.

With Eq. (17) and a body fitted surface in place, the natural validation target is the tabulated NACA 1- series cowl pressure data of Re and Abeyounis [11], which provides surface C_p distributions over a range of mass flow ratios for geometries whose ordinates are published in full. That comparison is the intended subject of the next stage of this work.

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9. Conflict of Interest

The author declares no competing conflict of interest.

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