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Generalized Solutions of Biquaternionic Form of Maxwell Equations for Moving Ball and Spherical Emitters

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Abstract: Transport solutions of the biquaternion wave equation, which is a biquaternion generalization of Maxwell's equations describing the electro-gravimagnetic (EGM) fields of EGM wave emitters are constructed which are moving in a certain direction with a constant speed that is less than the speed of light. Fundamental and generalized transport solutions of the biwave equation for describing the fields of moving EGM wave emitters are constructed using the theory of generalized functions. The Green bifunction is constructed which describes the field of a concentrated charge-mass, and its energy-momentum biquaternion is investigated. Biquaternions of the EGM field strength of moving spherical and ball emitters are constructed. Use the obtained results to study the electromagnetic fields of various EM emitters and radio wave emitters located on moving objects (trains, cars, ships, etc.).

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1. Introduction

In the works [1-4] one biquaternionic model of the electro-gravimagnetic field and EGM interactions has been developed. It is based on biquaternion generalizations of the systems of Maxwell and Dirac equations, which have the form of biquaternion wave equations on the Minkowski space [5-7]. It is known that these equations of quantum mechanics are used to describe elementary particles, spinors and spinor fields [8-12]. Here, transport solutions of the biquaternion wave equation are constructed and studied, which is a biquaternion generalization of Maxwell's equations and describes the electro-gravimagnetic fields of emitters of electro-gravimagnetic waves moving with a constant speed in a fixed direction. Fundamental and generalized transport solutions are constructed that describe the fields of moving objects with speeds less than the propagation speed of electromagnetic waves in the medium. Since the algebra of biquaternions is not very well known, we will first give several definitions so as not to refer the reader to [1]. Here we use the representation of biquaternions in the form proposed by Hamilton [6], which is very clear and convenient for physical applications.

2. Biquaternions algebra

At first, we give some definitions of biquaternions algebra because it is almost unknown. Biquaternions space $\mathbf{B} = \{\mathbf{F} = f + F\}$ is the space of hyper complex numbers, where f is a complex number, F is a 3D vector with complex components: $F = \sum_{j=1}^{3} F_j e_j$; e_1, e_2, e_3 are the unit vectors of Cartesian coordinate system in R^3, $e_0 = 1$. **B** is a linear space with the addition: for any complex number a, b

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$$a\mathbf{F}+b\mathbf{G}=a(f+F)+b(g+G)=(af+bg)+(aF+bG),$$

and the operation of quaternion multiplication ($\circ$):

$$\mathbf{F}\circ\mathbf{G}=(f+F)\circ(g+G)=(fg-(F,G))+(fG+gF+[F,G]). \quad (1)$$

Here and further $(F,G)=\sum_{j=1}^{3}F_jG_j$ is scalar production F and G, $[F,G]=\sum_{j=1}^{3}\varepsilon_{jkl}F_jG_ke_l$ is their vector production, ε_{jkl} is Levi-Civita pseudo tensor, δ_{jk} is Kronecker symbol. The algebra of biquaternions is non-commutative because

$\mathbf{F}\circ\mathbf{G}-\mathbf{G}\circ\mathbf{F}=2[G,F]$ but it's associative:

$$\mathbf{F}\circ\mathbf{G}\circ\mathbf{H}=(\mathbf{F}\circ\mathbf{G})\circ\mathbf{H}=\mathbf{F}\circ(\mathbf{G}\circ\mathbf{H}) \quad (2)$$

Properties of biquaternions and many definitions considered in [1]. Let us list some of them.
Biquaternions commute, i.e. $\mathbf{F}\circ\mathbf{G}=\mathbf{G}\circ\mathbf{F}$, only if their vector parts are parallel $G \parallel F$, or at least one of them is equal to a scalar.
Definition 1. *Scalar multiplication* of biquaternions $\mathbf{F}_1,\mathbf{F}_2$ is the bilinear operation: $(\mathbf{F}_1,\mathbf{F}_2)=f_1f_2+(F_1,F_2)$.

Definition 2. *A norm of* a biquaternion $\mathbf{F}$ is equal to $\|\mathbf{F}\|=\sqrt{(\mathbf{F},\bar{\mathbf{F}})}=\sqrt{f\cdot\bar{f}+(F,\bar{F})}=\sqrt{|f|^2+\|F\|^2}$

Definition 3**.** *A pseudodonorm* of a biquaternion $\mathbf{F}$ is equal to $\langle\mathbf{F}\rangle=\sqrt{f\cdot\bar{f}-(F,\bar{F})}=\sqrt{|f|^2-\|F\|^2}$

3. Bigradients and biwave equations

We consider the functional space $\mathbf{B}(\mathbf{M})=\{\mathbf{F}(\tau,x)=f(\tau,x)+F(\tau,x)\}$ of biquaternions (Bqs). Here $f(\tau,x)$ is a complex-valued functions on Minkowski space $\mathbf{M}=\{(\tau=ct,x),\tau\in R^1,x\in R^3\}$, $F(\tau,x)$ is three-dimensional vector function with complex components ($F=F_1+iF_2$); f and F from the class of generalized functions of slow growth **[13]**.

Definition 4. *Mutual bigradients* are differential biquaternion operators of the form: $\nabla^+=\partial_\tau+i\nabla$, $\nabla^-=\partial_\tau-i\nabla$, where $\nabla=\mathrm{grad}=(\partial_1,\partial_2,\partial_3)$. Their action on $\mathbf{B}(\mathbf{M})$ is defined according to the quaternion multiplication:

$$\nabla^{\pm}\mathbf{F}=(\partial_\tau\pm i\nabla)\circ(f+F)=(\partial_\tau f\mp i(\nabla,F)\pm i\nabla f\pm\partial_\tau F\pm i[\nabla,F].$$

Here $(\nabla,F)=\mathrm{div}\,F$, $[\nabla,F]=\mathrm{rot}\,F$ (everywhere in double signs the upper or lower signs are implied).
Their superposition has a remarkable property that is easy to prove.

Lemma 1. *Composition of mutual bigradients is commutative and equal to*

$$\nabla^-(\nabla^+\mathbf{F})=\nabla^+(\nabla^-\mathbf{F})=(\nabla^-\circ\nabla^+)\mathbf{F}=\Box\mathbf{F}, \quad (3)$$

where $\Box=\dfrac{\partial^2}{\partial\tau^2}-\Delta$ *is the wave operator,* Δ *is the Laplace operator*

Using this lemma, it is easy to solve differential biquaternionic wave equation (*biwave* Eq):

$$\nabla^+\mathbf{A}=\mathbf{\Theta}(\tau,x). \quad (4)$$

Its solutions and invariance properties under Poincare-Lorentz transformations discussed in detail in **[4, 14]**.

It has been shown [1, 6, 11, 12] that this equation is a biquaternion generalization of Maxwell equations if

$$\mathbf{A}=\alpha+\sqrt{\varepsilon}E+i\sqrt{\mu}H, \quad \mathbf{\Theta}(\tau,x)=-\left(\frac{j^E}{\sqrt{\varepsilon}}+i\,\frac{j^H}{\sqrt{\mu}}\right). \tag{5}$$

Here $E(\tau,x), H(\tau,x)$ are the vectors of electric and magnetic intensity of electromagnetic (EM) field, j^E, j^H are the density of electric and magnetic currents, ε, μ are the electrical conductivity and magnetic permeability of the medium, $c=1/\sqrt{\varepsilon\mu}$ is the *light speed* in a medium. When $\alpha=0,\ j^H=0$ Eq (5) is equivalent to the system of eight Maxwell equations. Here we consider transport solutions of Eq (4) without these restrictions.

4. Transport biwave equation and its general solution

Let us consider the case when the right side of (4) has the form:

$$\mathbf{\Theta}(\tau,x)=\mathbf{F}(x_1,x_2,z), \qquad z=x_3-\mathrm{M}\tau. \tag{6}$$

Here is $\mathbf{F}(x,z)$ describes the movement of the emitter in the direction of the axis X_3 with the speed v=*Mc*. There are three possible cases: *subluminal* $M<1 \Leftrightarrow \mathrm{v}<c$, *luminal* $M=1 \Leftrightarrow \mathrm{v}=c$, *superluminal* $M>1 \Leftrightarrow \mathrm{v}>c$, which change the type of this equation the type of its solutions.

Let us introduce a moving coordinate system (x_1,x_2,z) and construct solution (4) in a similar form $\mathbf{A}=\mathbf{B}(x,z)$, $x=(x_1,x_2)$, then $\partial_\tau=-\mathrm{v}\partial_z$, $\nabla=(\partial_1,\partial_2,\partial_z)$ and in the moving coordinate system Eq(4) is transformed to the form:

$$M_{\mathrm{v}}^{+}\mathbf{B}=\mathbf{F}(x,z), \tag{7}$$

$M_{\mathrm{v}}^{+} \triangleq -M\partial_z+i(\partial_1,\partial_2,\partial_z)$. We name this equation the *transport Maxwell equation* (TME).

Lemma 2 follows from Lemma 1.

Lemma 2. *The composition of mutual transport Maxwell operators is commutative and equal to*

$$M_{\mathrm{v}}^{\pm}M_{\mathrm{v}}^{\mp}=-\left\{\Delta_2+\left(1-M^2\right)\partial_{zz}\right\},$$

where $\Delta_2=\partial_1^2+\partial_2^2$ *- two-dimensional Laplacian.*

By using this lemma, we prove the next theorem.

Theorem 1. *The general solution of MTE can be represented in form*

$$\mathbf{B}(x,z)=\mathbf{B}^0(x,z)-M_{\mathrm{v}}^{\mp}\circ\left(\mathbf{F}(x,z)*\psi(x,z)\right), \tag{8}$$

where $\mathbf{B}^0(x,z)$ *is the solution of the homogeneous equation* (8) *(at* $\mathbf{F}=0$*),* $\psi(x,z)$ *is the fundamental solution of the transport wave equation*:

$$\Delta_2\psi+\mu^2\partial_{zz}\psi+\delta(z)\delta(x)=0, \tag{9}$$

where $\mu^2=1-M^2$, $\delta(\ldots)$ *is singular delta-function.*

Proof. Let's substitute it into equation (8) and, using the conditions of the theorem, lemma 2, the associativity property of quaternion multiplication and the properties of convolution with the delta function, we obtain the required:

$$M_{\rm v}^{\pm} \circ \left\{ \mathbf{B}^0 - M_{\rm v}^{\mp} \circ \left(\mathbf{F} * \psi \right) \right\} = M_{\rm v}^{\mp} \mathbf{B}^0 - M_{\rm v}^{\pm} \circ M_{\rm v}^{\mp} \circ \left(\mathbf{F} * \psi \right) = \left\{ \Delta_2 + \mu^2 \partial_{zz} \right\} \psi * \mathbf{F} = \delta(z)\delta(x) * \mathbf{F} = \mathbf{F}$$

It remains to calculate the *scalar potential* $\psi(x,z)$. Its form depends on the sign $\mu^2 = 1 - {\rm v}^2$:

$$\text{if } M<1 \Rightarrow \mu^2 > 0, \ \text{ if } M=1 \Rightarrow \mu^2 = 0, \ \text{if } M>1 \Rightarrow \mu^2 < 0.$$

Here we construct a solution of TME in subluminal case at $M<1$.

5. Sublight transport solutions of Maxwell's equation

In this case, to construct the original, we use the fundamental solution of the Laplace equation: $\Delta U + \delta(y) = 0, \quad y = (y_1, y_2, y_3)$, which has the form **[13]**:

$$U(y) = \frac{1}{4\pi \|y\|}. \tag{10}$$

Let us designate $z' = z/\mu$. Because

$$\partial_1^2 \psi + \partial_2^2 \psi + \mu^2 \partial_z^2 \psi + \delta(x,z) = 0 \Rightarrow \partial_1^2 \psi + \partial_2^2 \psi + \partial_{z'}^2 \psi + \mu^{-1} \delta(x)\delta(z') = 0,$$

then

$$\psi(x,z) = \frac{1}{4\pi\sqrt{z^2 + (\mu r)^2}}, \quad r = \|x\|.$$

Theorem 2. *The general solution of transport Maxwell equation* (7) *at subluminal speeds can be represented as*

$$\mathbf{B}(x,z) = M_{\rm v}^{-}\left(\mathbf{F} * \psi(x,z) \right) + M_{\rm v}^{-}\left(\psi_0(x,z) * \mathbf{C}(x,z) \right) \tag{11}$$

where scalar potentials are defined by the following formulas:

$$\psi(x,z) = \frac{1}{4\pi\sqrt{z^2 + (\mu r)^2}}, \tag{12}$$

$\psi_0(x,z)$ *is a solution of a homogeneous equation*

$$\Delta_2 \psi_0 + \mu^2 \partial_{zz} \psi_0 = 0, \tag{13}$$

$\psi_0 = \psi_0(x, z/\mu)$ *is a harmonic function,* $\mathbf{C}(x,z)$ is an *arbitrary biquaternion that admits convolution with* $\psi(x,z)$.

When differentiating, one should use the property of differentiation of the convolution in formula (14), which allows one to transfer differentiation to a more convenient component of the convolution, considering

$$\partial_z \psi = -\frac{z}{4\pi\sqrt{(z^2 + (\mu r)^2)^3}}, \quad \partial_k \psi = -\frac{\mu^2 x_k}{4\pi\sqrt{\left(z^2 + (\mu r)^2\right)^3}}, \quad k = 1,2. \tag{14}$$

Figure 1 shows a graph of the change in potential along the z-axis depending on the Mach numbers.

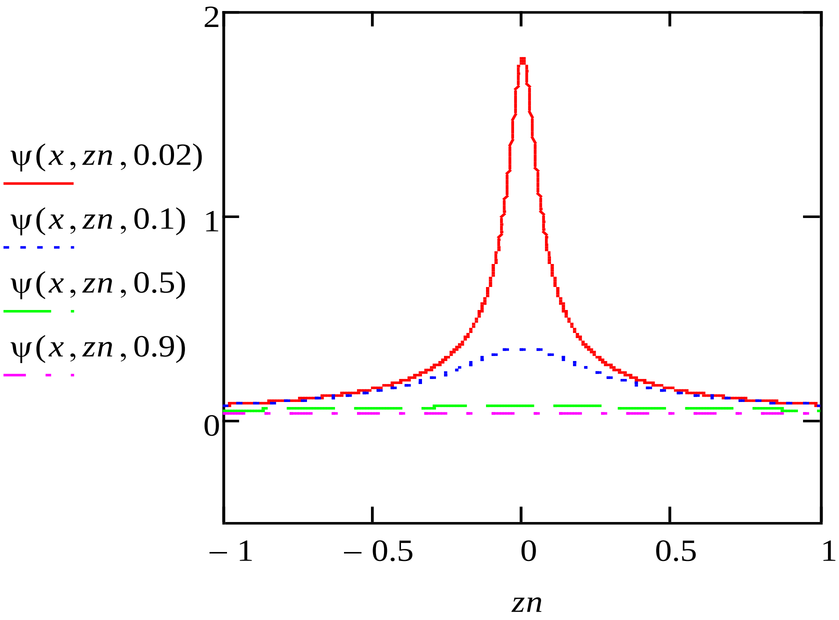

Figure-1 Change $\psi(x,z)$ along the Z axis at fixed *x*= (1,1) at different Mach numbers: *M*=0.02, 0.1, 0.5, 0.9

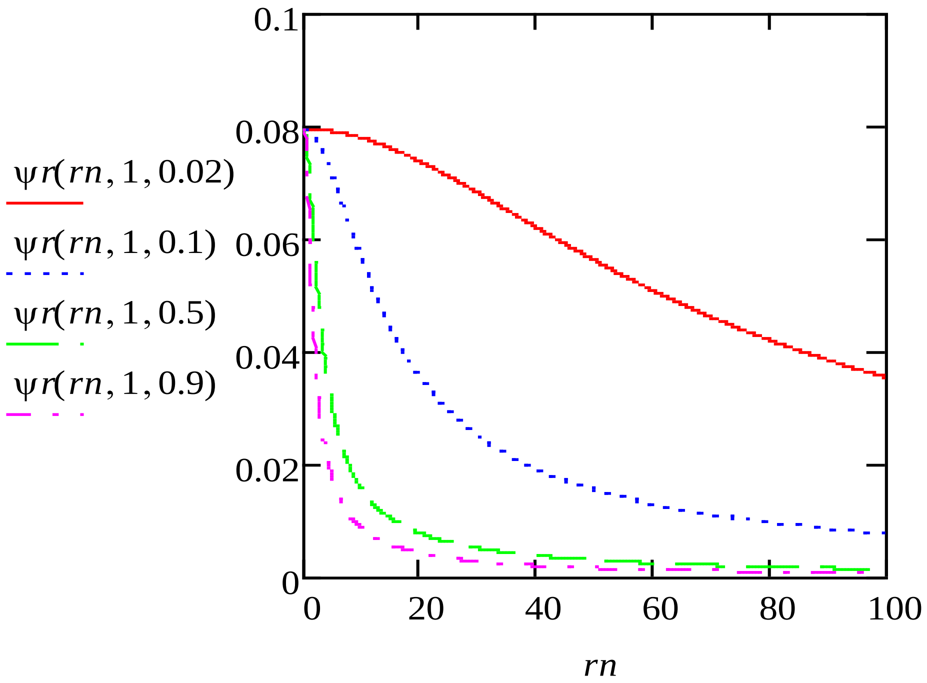

Figure-2 Change $\psi(x,z)$ along the *r* at fixed *z*=1 by different Mach numbers: *M*=0.02, 0.1, 0.5, 0.9.

6. Green bifunction and generalized solutions of transport Maxwell equation

Definition 6. *Green bifunction* $\mathbf{U}(x,z)$ is the fundamental solution of the equation (7):

$$M_v^+\mathbf{U} = \delta(x)\delta(z), \tag{15}$$

which satisfies the attenuation conditions at infinity:

$$\|\mathbf{U}(x,z)\| \to 0 \quad at\ \|(x,z)\| \to \infty\ . \qquad (16)$$

Bifunction $\mp\mathbf{U}(x,z)$ describe EGM field of a moving concentrated charge (electron or positron, accurate to the sign). Bifunction $i\mathbf{U}(x,z)$ describes EGM field of a moving material point which mass is equal to 1.

It is convenient to use Green bifunction for construction the solutions of Eq (7).
From Theorem 1 we get its form:

$$\mathbf{U}(x,z) = M_{\mathbf{V}}^{-} * \psi(x,z) = -\frac{Mz + i(\mu^2 r,_1, \mu^2 r,_2, z)}{4\pi\sqrt{(z^2+(\mu r)^2)^3}}, \quad r,_j = \frac{x_j}{r} \qquad (17)$$

Theorem 3. *General solution of TME* (7) *at subluminal speeds of motion, satisfying the attenuation conditions at infinity*: $\mathbf{B}(x,z) \to 0,\quad \|(x,z)\| \to \infty$, *has the form of biquaternionic convolution:*

$$\mathbf{B}(x,z) = \mathbf{U}(x,z) * \mathbf{F}(x,z) \qquad (18)$$

The solution exists for any $\mathbf{F}(x,z)$ *that allow such a convolution.*

Proof.

$$M_{\mathrm{v}}^{-}\mathbf{B}(x,z) = M_{\mathrm{v}}^{-} \circ (\mathbf{U} * \mathbf{F}(x,z)) = \left(M_{\mathrm{v}}^{-}\mathbf{U}\right) * \mathbf{F}(x,z) = \delta(x)\delta(z) * \mathbf{F}(x,z) = \mathbf{F}(x,z)\,.$$

If $\mathbf{F}(x,z)$ is a *regular* biquaternion, then formula (18) can be represented in the following integral form:

$$\mathbf{B}(x,z) = \mathbf{U} * \mathbf{F}(x,z) = \int_{-\infty}^{\infty} d\zeta \iint_{R^2} \mathbf{U}(x-y, z-\zeta) \circ \mathbf{F}(y,\zeta) dy_1 dy_2 \qquad (19)$$

If $\mathbf{F}(x,z)$ is a *simple layer* on the surface *S*: $\mathbf{F}(x,z) = \mathbf{C}(x,z)\delta_S(x,z)$ then

$$\mathbf{B}(x,z) = \mathbf{U} * \mathbf{C}(x,z)\delta_S(x,z) = \int_S \mathbf{U}(x-y, z-\zeta) \circ \mathbf{C}(y,\zeta) dS(y,\zeta)\ . \qquad (20)$$

Here the integral over the surface S.

If $\mathbf{F}(x,z)$ is a *simple layer on a curve L*: $\mathbf{F}(x,z) = \mathbf{C}(x,z)\delta_L(x,z)$ then formula (18) is the integral along the curve L:

$$\mathbf{B}(x,z) = \mathbf{U} * \mathbf{C}(x,z)\delta_L(x,z) = \int_L \mathbf{U}(x-y, z-\zeta) \circ \mathbf{C}(y,\zeta) dL(y,\zeta)\ . \qquad (21)$$

Here the integral over the curved line L.

The densities of all layers must be integrable functions on the layer support.

If $\mathbf{F}(x,z)$ is essentially singular, for example has a point support, then the convolution in (17) should be taken according to the rules of convolution in the space of generalized functions.

7. Energy density and Pointing vector of the electromagnetic field of moving concentrated charge

The *energy-momentum* biquaternion $\mathbf{\Sigma}(x,z)$ is equal to

$$\mathbf{\Sigma}(x,z)=0,5\mathbf{B}(x,z)\circ\mathbf{B}^{*}(x,z)=w(x,z)+iP(x,z)=0,5\left\{(b(x,z),\bar{b}(x,z))+(B(x,z),\bar{B}(x,z))\right\}+$$
$$+0,5\left\{\bar{b}(x,z)B(x,z)-b(x,z)\bar{B}(x,z)-\left[B(x,z),\bar{B}(x,z)\right]\right\}.$$

Here $w(x,z)$, $P(x,z)$ is the energy density and the analogue of the Pointing vector, which shows the direction of its propagation.

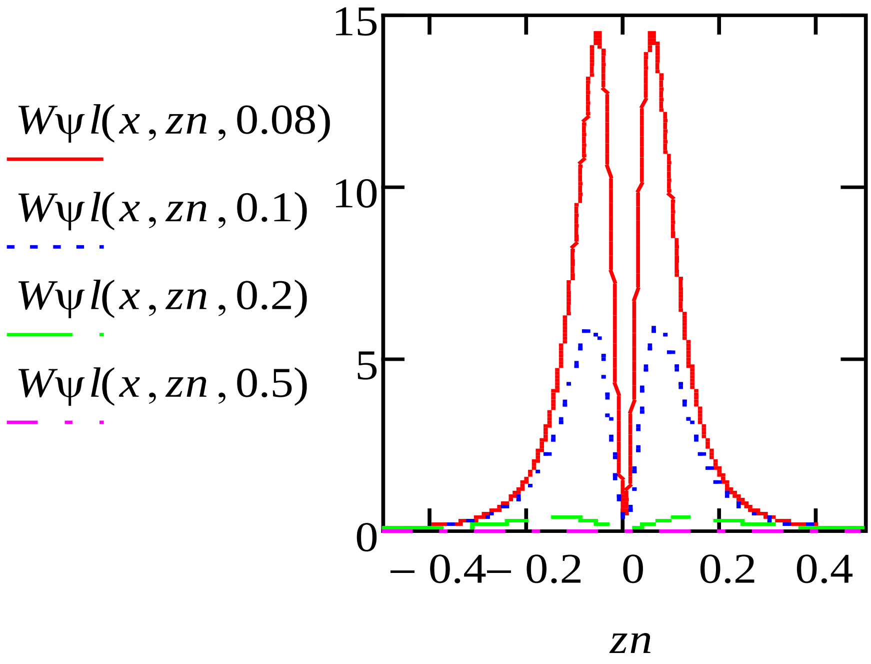

Figure-3 Energy density $w(x,z)$ along Z-axis by x = (1,1): M=0.08, 0.1, 0.2, 0.5.

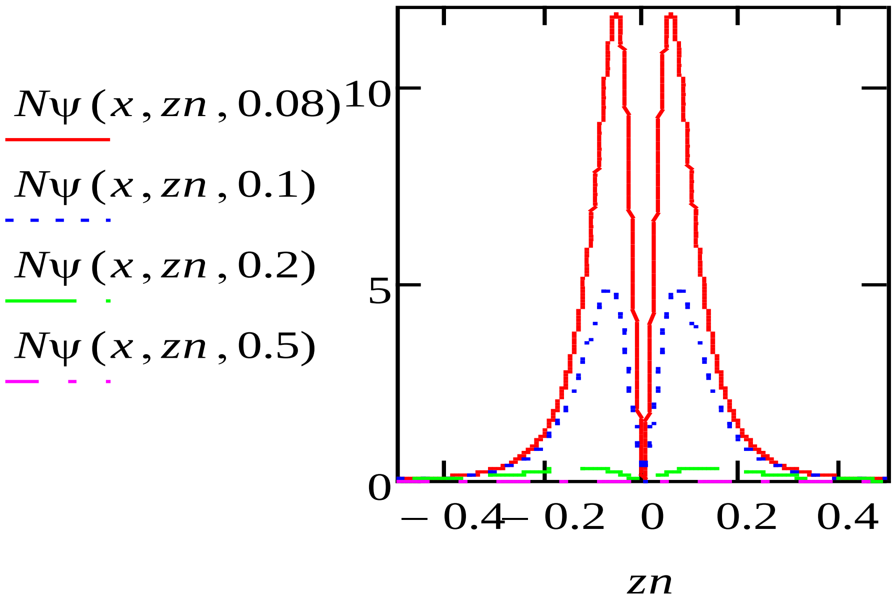

Figure-4 Norm of the Pointing vector at different Mach numbers: M=0.08, 0.1, 0.2, 0.5.

We define the energy-momentum biquaternion of the EM field of the Green bifunction:

$$\boldsymbol{\Sigma}_{\mathbf{U}}(x,z)=0,5\mathbf{U}(x,z)\circ\mathbf{U}^{*}(x,z)=\left(-M\frac{\partial\psi}{\partial z}+i\operatorname{grad}\psi\right)\circ\left(-M\frac{\partial\psi}{\partial z}-i\operatorname{grad}\psi\right),$$
$$w=0,5\left(M^{2}\left(\frac{\partial\psi}{\partial z}\right)^{2}+\left\|\operatorname{grad}\psi\right\|^{2}\right),\quad P=M\ \operatorname{grad}\psi\frac{\partial\psi}{\partial z}. \tag{21}$$

Using (17) and (21) we calculate the energy density and Pointing vector:

$$w(x,z)=\frac{1}{2}\left((M^{2}+1)\left(\frac{\partial\psi}{\partial z}\right)^{2}+\left(\frac{\partial\psi}{\partial x_{1}}\right)^{2}+\left(\frac{\partial\psi}{\partial x_{2}}\right)^{2}\right)=\frac{z^{2}(M^{2}+1)+\mu^{4}r^{2}}{32\pi^{2}(z^{2}+(\mu r)^{2})^{3}}$$

$$P(x,z)=\frac{Mz}{16\pi^{2}(z^{2}+(\mu r)^{2})^{3}}\left(\mu^{2}x_{1}e_{1}+\mu^{2}x_{2}e_{2}+ze_{3}\right),\qquad \left\|P(x,z)\right\|=\frac{M\left|z\right|\sqrt{z^{2}+\mu^{4}r^{2}}}{16\pi^{2}(z^{2}+(\mu r)^{2})^{3}}$$

At figure 3 we see the change in energy density $w(x,z)$ along Z-axis by x = (1,1) at different Mach numbers. There are two density maxima behind and ahead of the moving density source, near it, as the distance from the source increases, the energy density quickly fades. With decreasing speed of movement, the maxima increase, become sharper and approach the emitter. The Pointing vector has a similar behavior.

As an example, we will construct here biquaternions of moving spherical and ball emitters and their fields.

8. The intensity of the electro-gravimagnetic field of moving spherical emitter

Biquaternion of moving spherical emitters is a simple layer on the sphere S_b with radius b:

$$\mathbf{F}(x,z)=\mathbf{\Theta}(x,z)\delta(b-\|x\|)\,. \tag{22}$$

The intensity of its EGM field is equal to

$$\mathbf{B}(x,z)=\mathbf{U}*\mathbf{\Theta}(x,z)\delta_{S}(x,z)=\int\limits_{\|(y,\zeta)\|=b}\mathbf{U}(x-y,z-\zeta)\circ\mathbf{\Theta}(y,\zeta)dS(y,\zeta) \tag{23}$$

where $\mathbf{\Theta}(x,z)$ is integrable Bq on S_b. It is a density of a simple layer on S_b, which is defined acting charge-currents on it. Here we construct the biquaternions of the three types of moving spherical emitters

$$\mathbf{F}1=\rho(x,z)\delta(b-\|(x,z)\|),\quad \mathbf{F}2=d\delta(b-\|(x,z)\|)\left[e_{3},(x,z)\right],\quad \mathbf{F}2=d(x,z)\delta(b-\|(x,z)\|)\left[e_{1},(x,z)\right].$$

Biquaternions of moving charged sphere **is** $\mathbf{F}1(x,z)=\rho(x,z)\delta(b-\|(x,z)\|)$,

$$\mathbf{B}1(x,z)=\mathbf{U}*\rho(x,z)\delta(b-\|(x,z)\|)=\int\limits_{\|(y,\zeta)\|=b}\rho(x,z)\mathbf{U}(x-y,z-\zeta)\,dS(y,\zeta)$$

Biquaternions of moving sphere with rotating currents around e_3 is $\mathbf{F}2=J\left[e_{3},(x,z)\right]\delta(b-\|x\|)$,

$$\mathbf{B}2(x,z)=\mathbf{U}*d(x,z)\left[e_{3},(x,z)\right]\delta(b-\|(x,z)\|)=\int\limits_{\|(y,\zeta)\|=b}\mathbf{U}(x-y,z-\zeta)\circ\left(d(x,z)\left[e_{3},(y,\zeta)\right]\right)dS(y,\zeta)$$

Biquaternions of moving sphere with rotating currents around e_1 is $\mathbf{F}2=d\left[e_{1},(x,z)\right]\delta(b-\|x\|)$

$$\mathbf{B}3(x,z)=\mathbf{U}*d\left[e_1,(x,z)\right]\delta(b-\|(x,z)\|)=\int_{\|(y,\zeta)\|=b}\mathbf{U}(x-y,z-\zeta)\circ\left(d(x,z)\left[e_1,(y,\zeta)\right]\right)dS(y,\zeta)$$

The densities of all layers must be integrable functions on the layer support.

9. The intensity of the electro-gravimagnetic field of moving ball

Biquaternion of moving ball emitters with radius b is equal to

$$\mathbf{F}(x,z)=\mathbf{\Theta}(x,z)H(b-\|x\|). \tag{24}$$

The intensity of its EGM field is equal to

$$\mathbf{B}(x,z)=\mathbf{U}*\mathbf{\Theta}(x,z)H(b-\|x\|)=\int_{\|(y,\zeta)\|\le b}\mathbf{U}(x-y,z-\zeta)\circ\mathbf{\Theta}(y,\zeta)dV(y,\zeta), \tag{25}$$

where $\mathbf{\Theta}(x,z)$ is a density of charge-currents biquaternion in this ball. Here we construct the biquaternions of the three types of moving ball emitters:

$$\mathbf{F}1=\rho H(b-\|x\|),\quad \mathbf{F}2=d\left[e_3,(x,z)\right]H(b-\|x\|),\quad \mathbf{F}2=d\left[e_1,(x,z)\right]H(b-\|x\|).$$

Biquaternions of moving charged ball is equal to $\mathbf{F}1(x,z)=\rho(x,z)\delta(b-\|(x,z)\|)$,

$$\mathbf{B}1(x,z)=\mathbf{U}*\rho(x,z)H(b-\|(x,z)\|)=\int_{\|(y,\zeta)\|\le b}\rho(y,\zeta)\mathbf{U}(x-y,z-\zeta)\,dV(y,\zeta).$$

Biquaternions of moving ball with rotating currents around e_3 is equal to $\mathbf{F}2=d(x,z)\left[e_3,(x,z)\right]H(b-\|(x,z)\|)$,

$$\mathbf{B}2(x,z)=\mathbf{U}*d(x,z)\left[e_3,(x,z)\right]H(b-\|(x,z)\|)=\int_{\|(y,\zeta)\|\le b}\mathbf{U}(x-y,z-\zeta)\circ\left(d(y,\zeta)\left[e_3,(y,\zeta)\right]\right)dV(y,\zeta)$$

Biquaternions of moving ball with rotating currents around e_1 is equal to $\mathbf{F}2=d\left[e_1,(x,z)\right]H(b-\|(x,z)\|)$

$$\mathbf{B}3(x,z)=\mathbf{U}*d(x,z)\left[e_1,(x,z)\right]H(b-\|(x,z)\|)=\int_{\|(y,\zeta)\|\le b}\mathbf{U}(x-y,z-\zeta)\circ\left(d(y,\zeta)\left[e_1,(y,\zeta)\right]\right)dV(y,\zeta)$$

The densities of all charge-currents are integrable functions in this ball.

10. Conclusion

If in formulas (22)-(26) we take in the form (5) when, we obtain solutions of Maxwell's equations, which describe the electromagnetic fields of moving emitters of electromagnetic waves. For classical Maxwell's equations, they are constructed in **[14]**.

The obtained results be used to study the electromagnetic fields of various EM emitters and radio wave emitters located on moving objects (trains, cars, ships, etc.).

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12. Conflict of Interest

All presented results are new. The authors are not aware of similar works in this direction. There is no conflict of interest with other researchers.

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